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Non-factorization and the Decays $B \to J/\psi + K^{(*)}$

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Abstract

Many known models, which generally use a factorization hypothesis, give a poor account of the decays $B \to J/\psi + K^{(*)}$. Usually there is a free overall factor, which is fit to the data, so that tests of the models rely upon ratios. The models tend to give too much K^* compared to K and too much transverse polarization compared to longitudinal. Our microscopic calculations, which use perturbative QCD, do well for both ratios. A microscopic calculation allows us to see how well factorization, heavy quark symmetry, and other features of various models are working. In the present case, agreement with the experimental ratios is dependent upon a breakdown of factorization for one of the amplitudes.

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I. FACTORIZATION AND DATA

Gourdin, Kamal, and Pham [1] and Aleksan *et al.* [2] point out that many known models [3–6], all of which use the factorization hypothesis, give a poor account of the decays $B \to J/\psi + K^{(*)}$.

In most models there is an overall factor, generally called a_2 [3], which is fit to the data, so that tests of the models rely upon ratios of K^* and K decays, and ratios of longitudinal and tranverse polarization in the $J/\psi + K^*$ decays. The models tend to give too much K^* compared to K and too much transverse polarization compared to longitudinal.

Our microscopic calculations, which use perturbative QCD, do well for both ratios. Although the results have been published in some detail [7–9], the charmonium B decays deserve some further thought because of the present interest in them, and we will attempt to make self-contained at least the qualitative parts of our present remarks. A microscopic calculation allows us to see how well factorization, heavy quark symmetry, and other features of various models are working. In the present case, we find a serious breakdown of factorization for one of the amplitudes.

More explicitly, the ratios under study and their experimental values are [10],

$$R \equiv \frac{\mathrm{Br}(B \to J/\psi + K^*)}{\mathrm{Br}(B \to J/\psi + K)} = 1.71 \pm 0.40, \tag{1}$$

and, dividing the K^* rate into a longitudinal polarization part Γ_L and a transverse one Γ_T,

$$\left(\frac{\Gamma_L}{\Gamma}\right)_{K^*} = \begin{cases} 0.80 \pm 0.08 \pm 0.05 & \text{CLEO [10]} \\ 0.66 \pm 0.10 \pm 0.10 & \text{CDF [11]} \end{cases} \tag{2}$$

Our own results for the two ratios are 1.76 and 0.65, respectively (using Table IV of [7]).

Factorization implies that the decays depend upon a set of form factors for a current connecting B to $K^{(*)}$. As a benchmark—yes, we know the $K^{(*)}$ is light—the relations that heavy quark symmetry [12–14] implies among the form factors lead to

$$R = \frac{m_B^2 + 4m_{J/\psi}^2}{m_B^2} \approx 2.38 \tag{3}$$

and

$$\left(\frac{\Gamma_L}{\Gamma}\right)_{K^*} = \frac{m_B^2}{m_B^2 + 4m_{J/\psi}^2} \approx 0.42, \tag{4}$$

which, although they do not agree with the data, are not bad as a representation of many of the models. For information, in each combination $m_B^2 + 4m_{J/\psi}^2$, the m_B^2 comes from Γ_L and the $4m_{J/\psi}^2$ comes from Γ_T.

Why does our calculation work for the ratios when others do not? Most importantly, the factorization hypothesis fails. It does not fail uniformly. Its failure is significant only for the transverse polarization final state of $B \to J/\psi + K^*$. In this amplitude the nonfactorizable

contributions are about half the size and opposite in sign to the factorizable ones, which has roughly the effect of turning the "4" into a "1" in the previous equations, and giving decent agreement with the $(\Gamma_{LL}/\Gamma)_{K^*}$ data.

Also, surprisingly in this context, we find the heavy quark symmetry symmetry predictions for the form factors of the factorizable parts of the amplitude work surprisingly well. One might expect significant differences due to a nonperturbative cause, namely that the wave functions or distribution amplitudes of the K and K^* are different. A wave function difference at the origin is shown by data that gives unequal decay constants for the K and K^*, and the shapes of the two wave functions are also different. We use the distribution amplitudes for K and K^* worked out from QCD sum rules by Chernyak, Zhitnisky, and Zhitnitsky [15]. The upshot is that the form factors relative to the heavy quark symmetry predictions are good, and that small corrections and nonfactorizable contributions keep the two ratios from just being inverses of each other.

Some details will now come.

II. MORE DETAILED DISCUSSION

Factorization

We should state what factorization means in the context of $B \to J/\psi + K^{(*)}$. The relevant part of the effective Hamiltonian density is

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} \; \bar{s}\gamma_{\mu L} c \; \bar{c}\gamma_L^\mu b, \tag{5}$$

where $\gamma_L^\mu = \gamma^\mu(1-\gamma_5)$ and the matrix element we want is, generically,

$$M = \langle X, \psi | H_{eff} | B \rangle, \tag{6}$$

where $\psi = \eta_c, J/\psi, \ldots$.

The factorization hypothesis is that the charmed quarks which are created go into the ψ and, except for the weak interaction vertices, are unconnected to other quarks in the process. We also assume that the outgoing charmed quarks in the ψ have small transverse momentum relative to the direction of the ψ. If the factorization hypothesis is valid, one can show

$$M = -\frac{1}{N_c} \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} \; (\bar{c}\gamma_{\mu L} c)_\psi \; (\bar{s}\gamma_L^\mu b)_{B\to X}, \tag{7}$$

that is, the matrix element is a product of two hadronic factors,

$$(\bar{c}\gamma_{\mu L} c)_\psi \equiv \langle \psi | \bar{c}\gamma_{\mu L} c | 0 \rangle \tag{8}$$

and

$$(\bar{s}\gamma_L^\mu b)_{B\to X} \equiv \langle X | \bar{s}\gamma_L^\mu b | B \rangle. \tag{9}$$

Quantity N_c is the number of colors, and to allow for effects of mixing with other operators one usually replaces $(-1/N_c)$ with a constant a_2 which is determined by data.

Non-factorization

In Fig. 1, parts (a) and (d) correspond to the factorizable contributions, in the present context, and parts (b) and (c) to the nonfactorizable ones.

Upon first view, it is easy to believe that the nonfactorizable contributions are small. The gluon couples to two oppositely colored quarks that are nearly at the same point because of the W-exchange. Indeed, the largest parts of diagrams (b) and (c) cancel each other and the subleading $O(q_G)$, where q_G is the gluon momentum, terms give the surviving result. The pieces of Figs. 1 (b) and (c) from one weak vertex, through the J/ψ (whose polarization vector is ξ) including the gluon emmission vertex (γ^ν), and to the other weak vertex, have numerators that sum to

$$4m_{J/\psi}(1+\gamma_5)\left(\not{\xi}\,\gamma^\nu \not{q}_G - \not{q}_G\,\gamma^\nu \not{\xi}\right)(1-\gamma_5), \tag{10}$$

which does go to zero for gluons of long wavelength, or q_G going to zero. (The numerators of Figs. 1 (a) and (d) do not go to zero in the same limit.) However, the gluon momentum is not so small; in fact we argue that it is large enough that a perturbative calculation is plausibly valid. It supplies the momentum transfer needed by the light quark, which is of order $\bar{\Lambda}_B$, the part of the mass of the B meson carried by the light quark, which is about 500 MeV or a few times Λ_{QCD}.

However, for $B \to J/\psi + K$ and the longitudinal part of $B \to J/\psi + K^*$, there is further cancellation between the subleading parts of the two nonfactorizable diagrams. In contrast, they add for the transverse decay, so this nonfactorizable amplitude can get large. While the transverse $B \to J/\psi + K^*$ does require chirality violations, the ensuing suppression is of $O(m_{J/\psi}/m_B)$, which is not a decisive factor.

III. CLOSING REMARKS

We have seen, in one explicit calculation, how nonfactorizable contributions to $B \to J/\psi + K^{(*)}$ are significant and are crucial to giving agreement with data for the K^* to K ratio and the tranverse to longitudinal polarization ratio.

Ratios are used to test the models because in most models the overall rate is determined by a constant that is fit to the data. Our calculations using perturbative QCD also have trouble with the overall rate. At present, with the parameters we choose [7], we do a good job on non-color-suppressed decays such as $B \to D\pi$ but the overall rates are rather low compared to data for decays like $B \to J/\psi + K^{(*)}$.

We believe the use of pQCD is valid for high recoil decays of the B. This has been given better support by Akhoury, Sterman, and Yao [16]who consider Sudakov effects in pQCD calculations of B decays and show that they suppress contributions from end point regions where use of pQCD would be questionable.

It is possible that perturbative contributions are part but not all of what gives $B \to J/\psi + K^{(*)}$ decay. In this case, the details of our remarks will only be part of something larger, but the scenario can well stand: the factorizable contributions to $B \to J/\psi + K^{(*)}$ will not suffice to explain those decays, and nonfactorizable contributions will be crucial.

APPENDIX A: FACTORIZATION HYPOTHESIS

Here is a perturbative proof that the factorization hypothesis leads to the factored form of the amplitude. Take the J/ψ as an example. Factorization allows us to isolate the ψ piece,

$$\begin{aligned}
&\bar{s}_i\gamma_\mu(1-\gamma_5)\langle J/\psi|c_i\bar{c}_j|0\rangle\gamma^\mu(1-\gamma_5)b_j \\
&= \bar{s}_i\gamma_\mu(1-\gamma_5)\left[\frac{f_{J/\psi}}{2\sqrt{N_c}}\frac{\delta_{ij}}{\sqrt{N_c}}\frac{\not{\xi}(\not{q}+m_{J/\psi})}{\sqrt{2}}\right]\gamma^\mu(1-\gamma_5)b_j \\
&= -\frac{1}{N_c}\left[\sqrt{2}f_{J/\psi}m_{J/\psi}\xi_\alpha\right]\bar{s}\gamma_L^\alpha b,
\end{aligned} \tag{A1}$$

where q and ξ are the momentum and polarization vectors of the J/ψ and i and j are color indices. One needs to recognize

$$\langle J/\psi|\bar{c}\gamma_\alpha c|0\rangle = \sqrt{2}f_{J/\psi}m_{J/\psi}\xi_\alpha \tag{A2}$$

to complete the proof.

APPENDIX B: FACTORIZABLE AMPLITUDES

It is of some interest to see the behavior of the factorizable parts of the amplitudes. In practice, diagram 1(d) is quite small and we will give the results from Fig. 1(a). We use form factors $f_\pm$, $a_\pm$, g, and f as defined in ref. [14]. Neither f_- nor a_- enters when the decay involves the J/ψ. If desired, one may convert to ref. [3] definitions by ($q^2 \leftrightarrow m^2_{J/\psi}$ here),

$$\begin{aligned}
F_1 &= f_+ \\
F_0 &= \frac{q^2}{m_B^2-m_K^2}f_- + f_+ \\
V &= -(m_B+m_{K^*})g \\
A_1 &= (m_B+m_{K^*})^{-1}f \\
A_2 &= -(m_B+m_{K^*})a_+ \\
A_0 &= (2m_{K^*})^{-1}(f+(m_B^2-m_{K^*}^2)a_+ + q^2a_-)
\end{aligned} \tag{B1}$$

We keep the explicit $m_{K^{(*)}}$ mass terms although they turn out to have small effect (unless necessary in a definition).

Each form factor can be written like

$$f_+ = B\int_0^1 dy_1\,\tilde{\phi}_K\frac{(1-y_1)(a+by_1)}{y_1-r-i\eta}, \tag{B2}$$

where

$$B = \frac{16\pi\alpha_s f_B f_{K^{(*)}}}{3\epsilon_B(m_B^2+m_{K^{(*)}}^2-m_{J/\psi}^2)^2} \tag{B3}$$

for $\epsilon_B = \bar{\Lambda}_B/m_B$ and

$$r = \frac{2\epsilon_B m_B^2}{m_B^2 + m_{K^{(*)}}^2 - m_{J/\psi}^2}. \tag{B4}$$

Quantity $\tilde{\phi}_K$ is the distribution amplitude of the kaon with the asyptotic form factored out (so that $\tilde{\phi}_K = 1$ if we wish to use the asymptotic form). For form factor f_+,

$$\begin{aligned} a = a(f_+) &= m_B(m_B + m_K) - \epsilon_B m_B(m_B - m_K) \\ b = b(f_+) &= m_B^2 - 2m_B m_K - m_{J/\psi}^2. \end{aligned} \tag{B5}$$

For the other form factors,

$$\begin{aligned} a(g) &= m_B(1 + \epsilon_B) \\ b(g) &= -m_{K^*} \end{aligned} \tag{B6}$$

and

$$\begin{aligned} a(f) &= 2m_B^2 m_{K^*}(1 - 2\epsilon_B) \\ &\quad +(m_B^2 + m_{K^*}^2 - m_{J/\psi}^2) m_B (1 + \epsilon_B) \\ b(f) &= m_{K^*}(m_B^2 + m_{K^*}^2 - 4m_B m_{K^*} - m_{J/\psi}^2). \end{aligned} \tag{B7}$$

It happens that the heavy quark symmetry results $a_\pm = \pm g$ are obeyed exactly.

The integrals can be done analytically, using

$$I_N = \int dy \, \frac{y^N}{y - r - i\eta} = \sum_{k=0}^{N-1} \frac{1}{N-k} r^k + r^N I_0, \tag{B8}$$

for $N \geq 1$ and

$$I_0 = i\pi + \ln\left(\frac{1-r}{r}\right). \tag{B9}$$

For asymptotic wave functions

$$\begin{aligned} &\frac{|f_+|}{m_B + m_K} : |g| : \frac{|f|}{(m_B - m_{K^*})^2 - m_{J/\psi}^2} \\ &\qquad = 0.99 f_K : 1.03 f_{K^*} : 0.99 f_{K^*}. \end{aligned} \tag{B10}$$

Heavy quark symmetry predicts 1:1:1. Of course, strict heavy quark symmetry also predicts $f_{K^*}/f_K = 1$ whereas the experimental result is $1.67 f_\pi / 1.22 f_\pi \approx 1.37$.

One should perhaps not use the asymptotic distribution amplitudes for the kaons. A common form for representing the distribution amplitude is

$$\tilde{\phi}(y_1) = 5\beta(2y_1 - 1)^2 + (1 - \beta), \tag{B11}$$

where y_1 is the momentum fraction carried by the nonstrange quark and β is the fraction of the distribution amplitude that is not asymptotic. The QCD sum rule results of Chernyak, Zhitnitsky, and Zhitnitsky, lead to $\beta = 0.6$ for the K and 0.1 for the K^*. One then gets

$$1.38 f_K : 1.03 f_{K^*} : 1.03 f_{K^*} \tag{B12}$$

for the same ratio as eqn. (B10) (with the same overall constants). This is stunningly close to the heavy quark symmetry result!

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FIGURES

FIG. 1. Lowest order perturbation theory diagrams for B decays involving charmonium. (a) and (d) are factorizable, (b) and (c) are nonfactorizable.